Lecture 2: Electron Emission and Cathode Emittance

- The objectives of this lecture are to define the basic electron emission statistics, describe the electrical potentials at the cathode surface, define the thermal emittance and derive the cathode emittance for thermal, photo-electric and field emission.
- This lecture begins with definitions of Maxwell-Boltzmann and Fermi-Dirac statistics, and discusses the electric fields at the cathode surface which the electron needs to overcome to escape. Then the physics of each of the three emission processes is described and their cathode emittances are derived.

Introduction

- The electron density inside a cathode is many orders of magnitude higher than that of the emitted electron beam.
- This is seen by considering that the density of conduction band electrons for metals is 10^{22} to 10^{23} electrons/cm^3. Whereas the density of electrons in a 6 ps long, 200 micron diameter cylindrical bunch with 1 nC of charge is ~10^{14} electrons/cm^3. Thus the transition from bound to free reduces the electron density by eight to nine orders of magnitude. In addition, the energy spread, or thermal energy of the electrons inside the cathode material is low. For example, in copper the energy spread near the Fermi energy is ~$k_B T$ or 0.02 eV at room temperature (300 degK). However, in order to release these cold, bound electrons, one needs to heat the cathode to approximately 2500 degK, resulting in a beam with a thermal energy of 0.20 eV.

Types of Electron Emission

- In general, the emission process determines the fundamental lower limit of the beam emittance. This ultimate emittance is often called the thermal emittance, due to the Maxwell-Boltzmann (MB) distribution of thermionic emitters. Strictly speaking, the term 'thermal emittance' should only be applied to thermionic emission, but the concept of thermal emittance or the intrinsic emittance of the cathode can be applied to the three forms of electron emission:
- 1. thermionic emission,
- 2. photo-electric emission
- 3. field emission.

Electron Statistics and the Emission Process

- Elementary particles in general can be classified as either bosons or fermions depending upon whether they have integer or half integer spin, respectively. Bosons obey classical Maxwell-Boltzmann statistics, while fermions follow Dirac-Fermi statistics. These statistics define the probability a particle occupies an given energy state based on the distribution of particles into energy intervals for the two particle types:

- 1. particles any number of which can share the same energy state, follow the Maxwell-Boltzmann distribution.

- 2. particles which cannot share the same energy state having only one particle per energy state, following the Fermi-Dirac distribution.

The Maxwell-Boltzmann and Fermi-Dirac Distributions

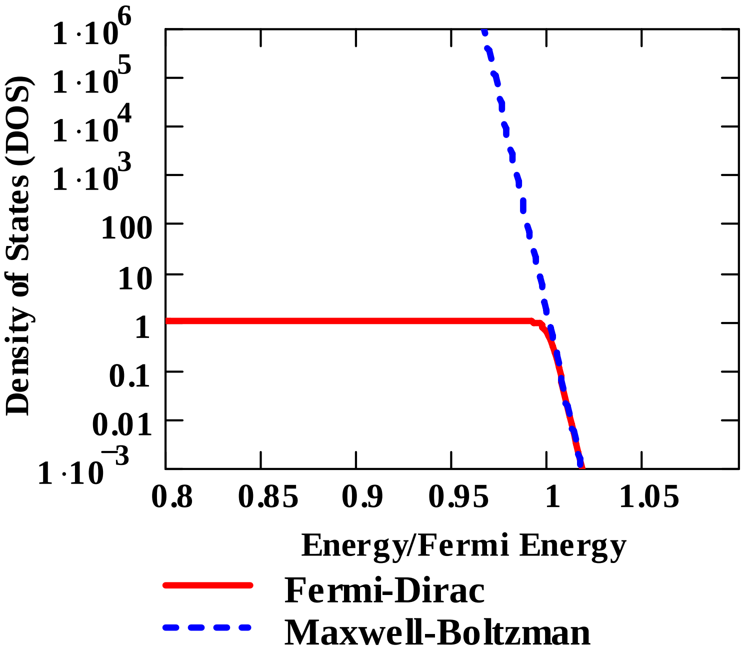
- The first particle type obeys classical Maxwell-Boltzmann (M-B) statistics with the energy distribution of occupied states given by,

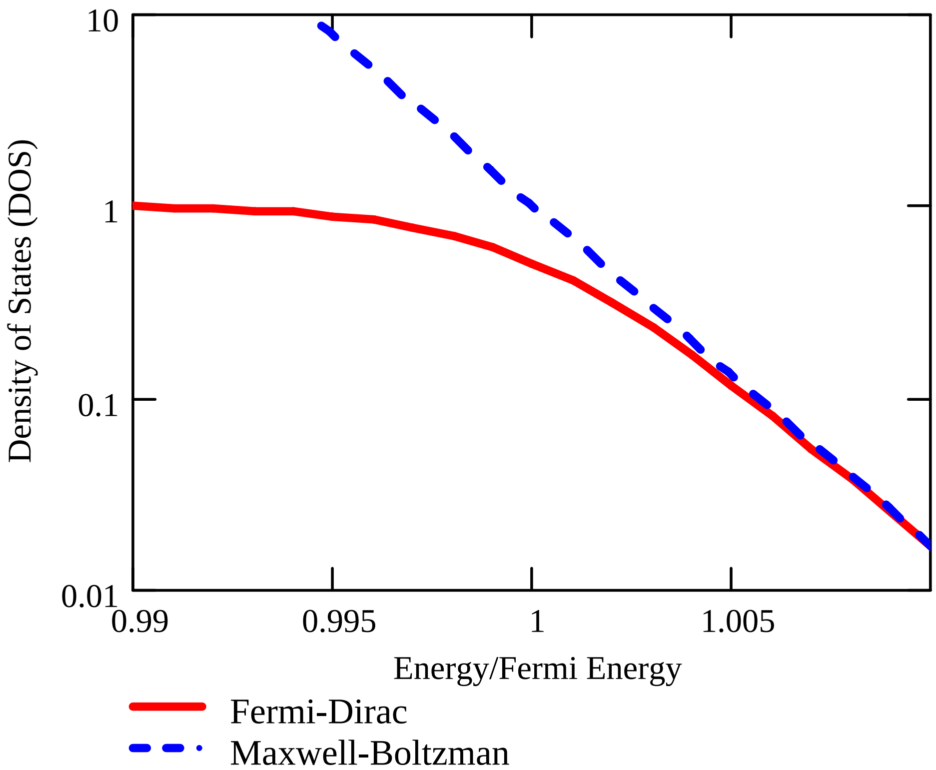
$$f_{MB} = e^{-E/k_B T}$$

- For the second particle type, of which electrons are a member being one-half spin fermions, the energy distribution of occupied states (DOS) is given by the Fermi-Dirac (F-D) function,

$$f_{FD} = \frac{1}{1 + e^{(E-E_F)/k_B T}}$$

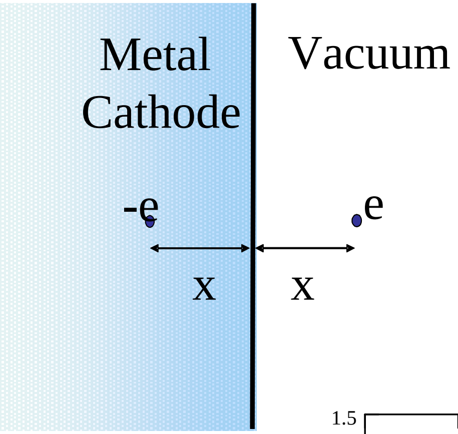
Comparison of M-B and F-D Distributions



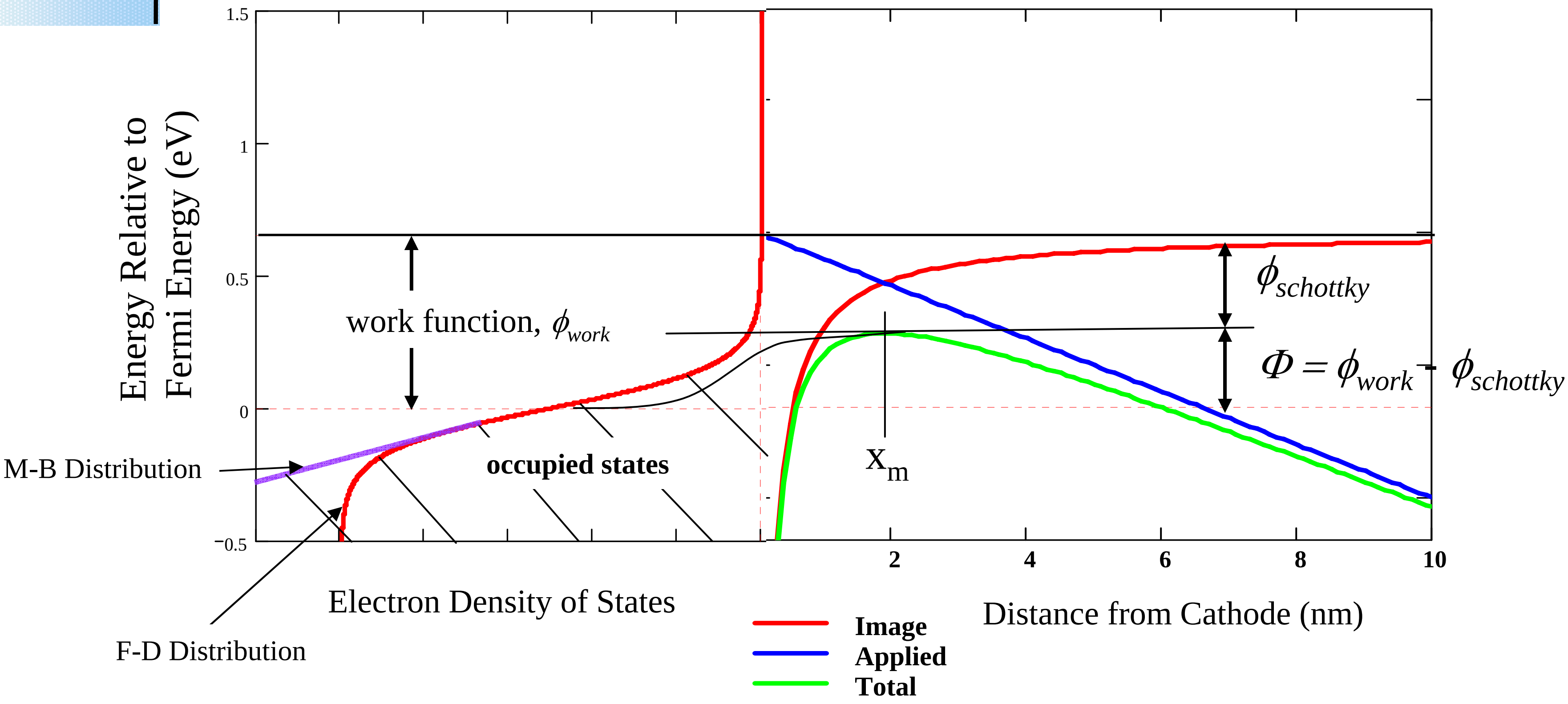


- Comparison of the particle energy distributions in the high-energy tails of the classical Maxwell-Boltzmann and the quantum mechanical Fermi-Dirac functions.

Fields Near the Cathode



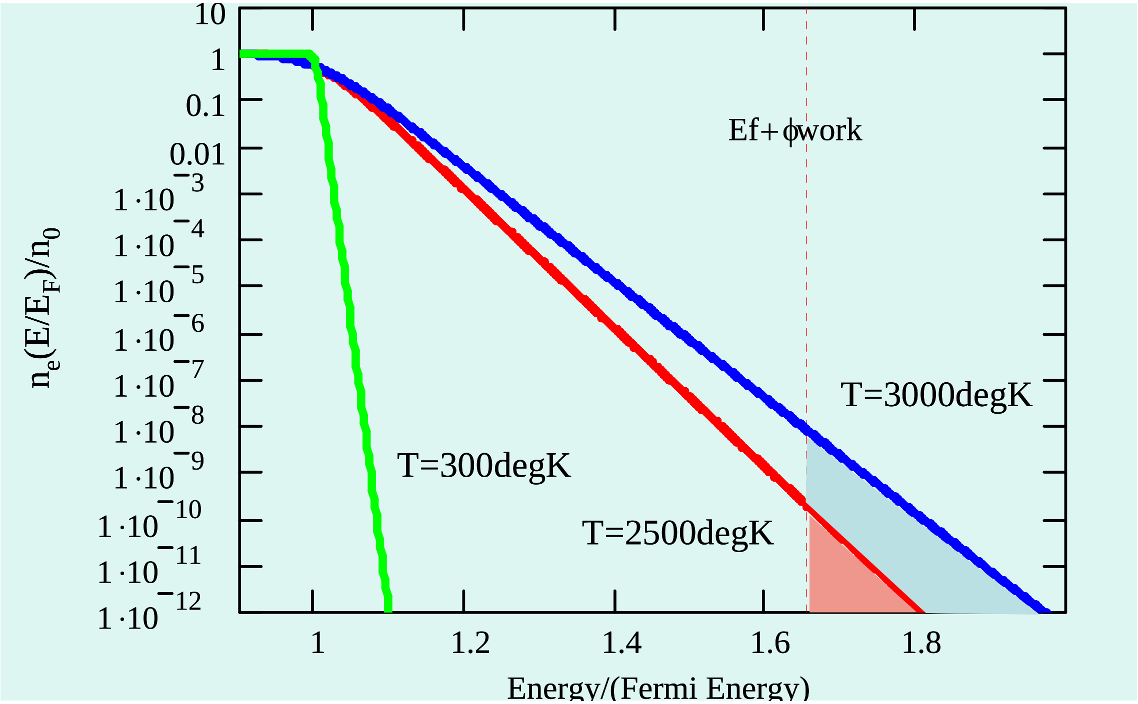
$$e\Phi = e\phi_{work} - \frac{e^2}{16\pi\epsilon_0 x} - eE_0 x$$



Thermionic Emission(1)

In order for an electron to escape a metal it needs to have sufficient kinetic in the direction of the barrier to overcome the work function,

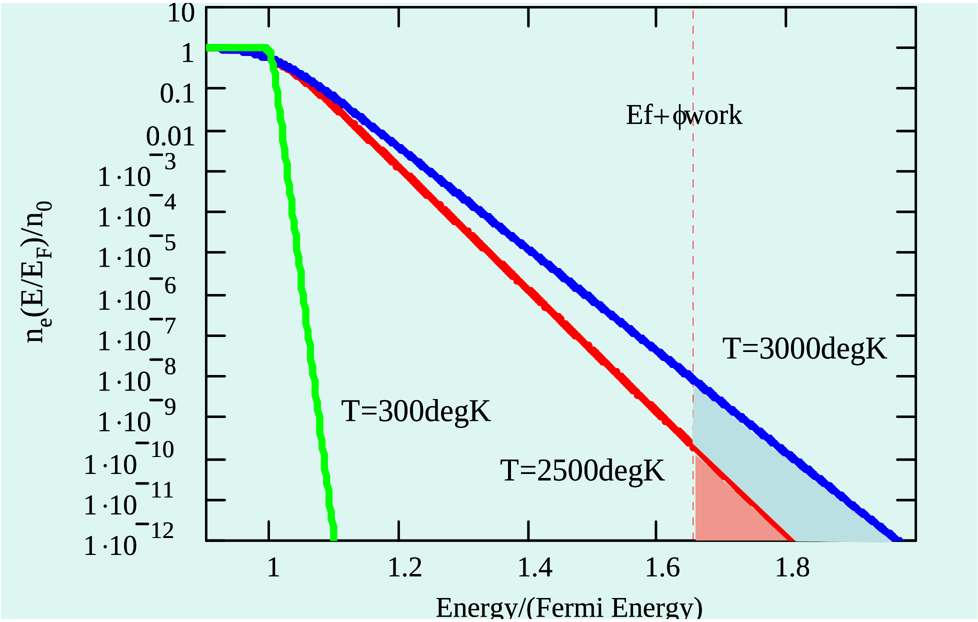
$$\frac{mv_x^2}{2} > e\phi_{work} \Rightarrow v_x > \sqrt{\frac{2e\phi_{work}}{m}}$$



Thermionic Emission(2)

- Assume that the cathode has an applied electric field large enough to remove all electrons from the surface, so there are no space charge effect, but low enough to not affect the barrier height. Then the thermionic current density for a cathode at temperature,

$$j_{thermionic} = n_0 e \langle v_x \rangle = n_0 e \int_{v_x > \sqrt{\frac{2e\phi_{work}}{m}}} v_x f_{FD} d\vec{v}$$



Thermionic Emission (3)

- As discussed above and shown in Figure 1, the interactions involving the high energy electrons in the tail of the Fermi-Dirac density of states allows it's replacement with the classical, Maxwell-Boltzmann distribution,

$$j_{thermionic} = n_0 e \int_{v_x > \sqrt{\frac{2e\phi_{work}}{m}}} v_x f_{MB} d\vec{v} = n_0 e \int_{v_x > \sqrt{2e\phi_{work}/m}} v_x e^{-\frac{m(v_x^2+v_y^2+v_z^2)}{2k_B T}} d\vec{v}$$

•Performing these simple integrals gives the thermionic current density,

$$j_{thermionic} = 2n_0 e \left(\frac{2k_B T}{m}\right)^2 e^{-\phi_{work}/k_B T}$$

Thermionic Emission(4)

- Or with a small change in the leading constants, gives the Richardson-Dushman equation for thermionic emission [Reiser, p 8],

$$j_{thermionic} = A(1-r)T^2 e^{-\phi_{work}/k_B T}$$

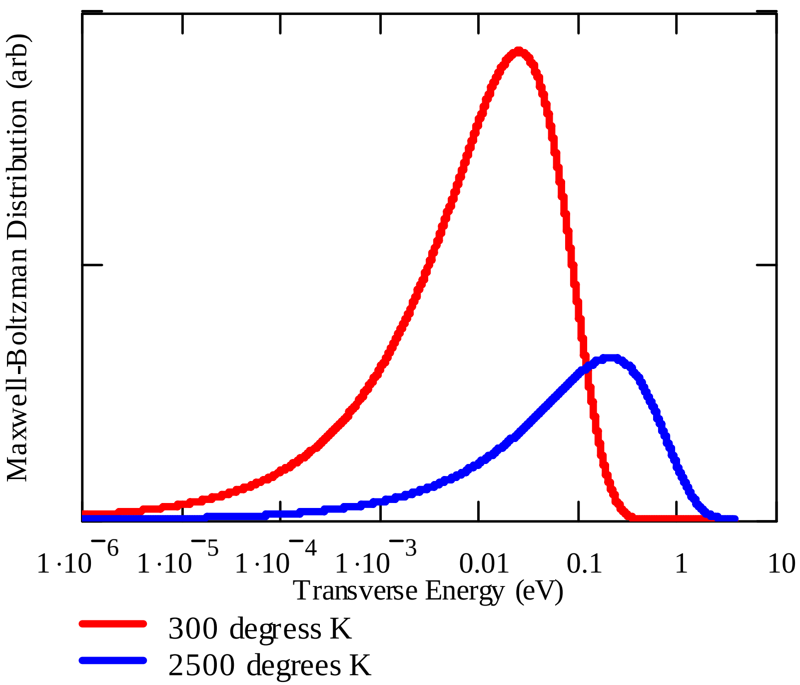
- Here A is 120 amp/cm^2/degK^2, and (1-r) accounts for the reflection of electrons at the metal surface. The reflection and refraction of electrons as they transit the surface is discussed in a later section. In terms of fundamental quantities, the universal constant A is ["Solid State Physics", by Ashcroft and Mermin, p. 363]

$$A = -\frac{em}{2\pi^2\hbar^3}$$

Thermionic Emittance (1)

- The velocity distribution for thermally emitted electrons is obtained from the derivative of Maxwell-Boltzmann particle distribution,

$$\frac{1}{n_e}\frac{dn(v_x)}{dv_x} = \frac{m}{k_B T} v_x e^{\frac{-m v_x^2}{2 k_B T}}$$



- **Maxwell-Boltzmann electron energy distributions at 300 degK where the rms electron energy spread is 0.049 eV, and at 2500 degK corresponding to an rms energy spread of 0.41 eV. The initial spread in transverse velocity due to the electron temperature gives the beam angular divergence and hence its thermionic emittance.**

Thermionic Emittance (2)

- Following Lawson [Lawson, p. 209], we assume the normalized emittance is evaluated close to the cathode surface where the electron flow is still laminar (no crossing of trajectories) and any correlation between position and angle can be ignored. In this case, normalized cathode emittance is given by,

$$\epsilon_N = \beta\gamma\sigma_x\sigma_{x'}$$

- The root-mean-square (rms) beam size, σ_x, is given by the transverse beam distribution which for a uniform radial distribution with radius R is R/2. The rms divergence is given by

$$\sigma_{x'} = \frac{\langle p_x \rangle}{p_{total}} = \frac{1}{\beta\gamma}\frac{\sqrt{\langle v_x^2 \rangle}}{c}$$

- The normalized, rms thermal emittance is then

$$\epsilon_n = \sigma_x \frac{\sqrt{\langle v_x^2 \rangle}}{c}$$

Reiser, p56-66

Thermionic Emittance (3)

- The mean squared transverse velocity for a M-B velocity distribution is,

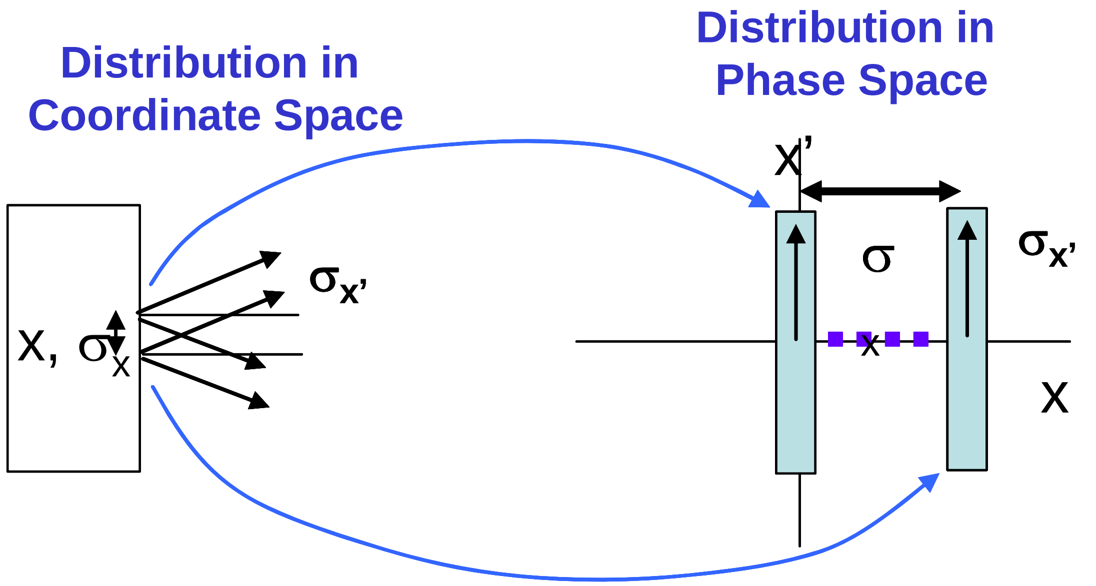
$$\langle v_x^2 \rangle = \frac{\int_0^\infty v_x^2 e^{-\frac{m v_x^2}{2 k_B T}} dv_x}{\int_0^\infty e^{-\frac{m v_x^2}{2 k_B T}} dv_x} = \frac{k_B T}{m}$$

- Therefore the thermionic emittance of a Maxwell-Boltzmann distribution at temperature, T, is

$$\epsilon_{thermionic} = \sigma_x \sqrt{\frac{k_B T}{m c^2}}$$

Thermionic Emittance (4)

- The divergence part of the cathode emittance contains all the physics of both the emission process and the cathode material properties and as such summarizes much of the interesting physics of the emission process. The beam size in coordinate space simply traces out the angular distribution to form the transverse phase space distribution as illustrated.



Thermionic Emission (5)

- Given that σ_x depends upon the particular transverse distribution being used, there is often a serious ambiguity which arises when expressing the thermal emittance in terms of "microns/mm". The confusion results in not knowing whether rms or flat top radii are used for the transverse radius. Therefore we suggest quoting a quantity called the normalized divergence, which for thermionic emission is

$$\Delta_{thermionic} \equiv \sqrt{\frac{k_B T}{mc^2}}$$

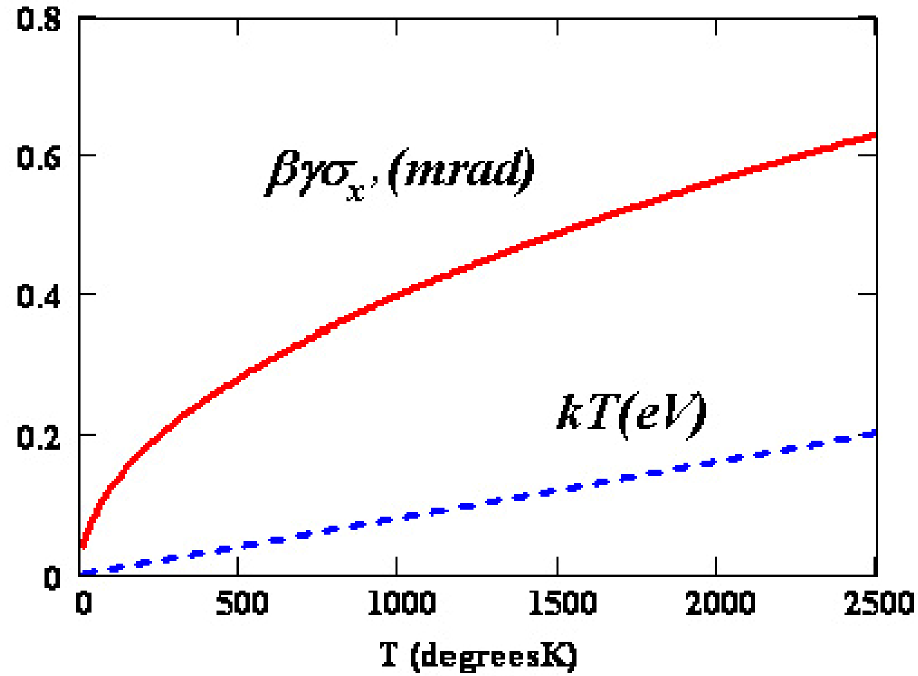


Photo-Electric Emission(1)

- Photoelectric emission from a metal can be described by the three steps of the Spicer model:
    1. Photon absorption by the electron
    2. Electron transport to the surface
    3. Escape through the barrier

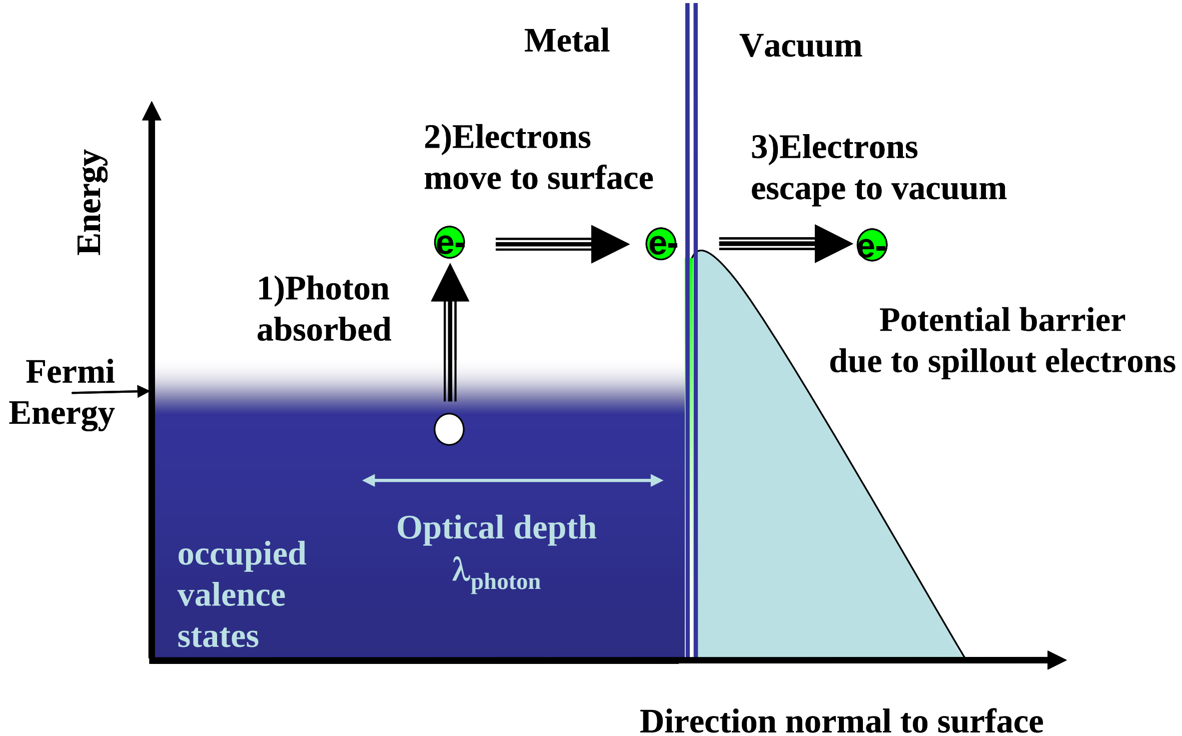


Photo-Electric Emission (2)

- Thermalization time of electrons in a metal as measured in a pump-probe experiment:

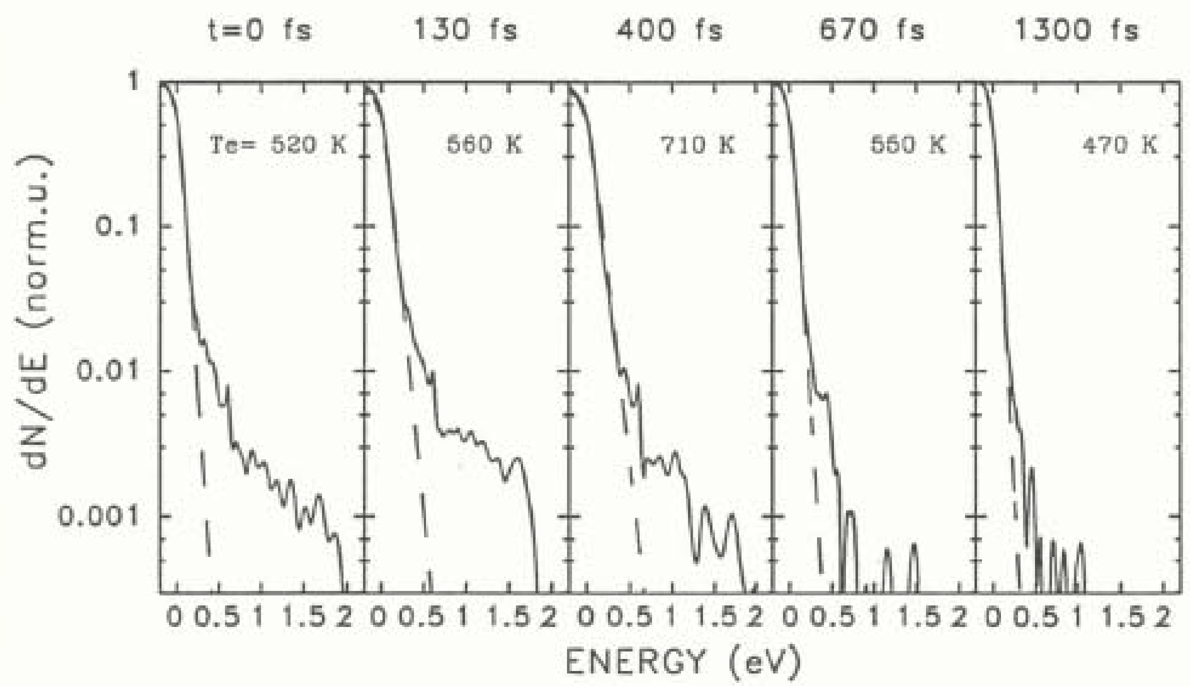


Fig. 1: Electron energy distribution function vs. energy with 120 μJ/cm^2 absorbed laser fluence at 5 time delays. The dashed line is the best Fermi-Dirac fit and the corresponding electron temperature, T_e, is shown. The vertical scale is in units of density of states.

Fann et al., "Observation of the thermalization of electrons in a metal excited by femtosecond optical pulses," in Ultrafast Phenomenona, ed. J.-L. Martin, A. Meigus, G.A. Mourou and A.H. Zewail, Springer Verlag 1993, p331-334.

Photo-Electric Emission (3)

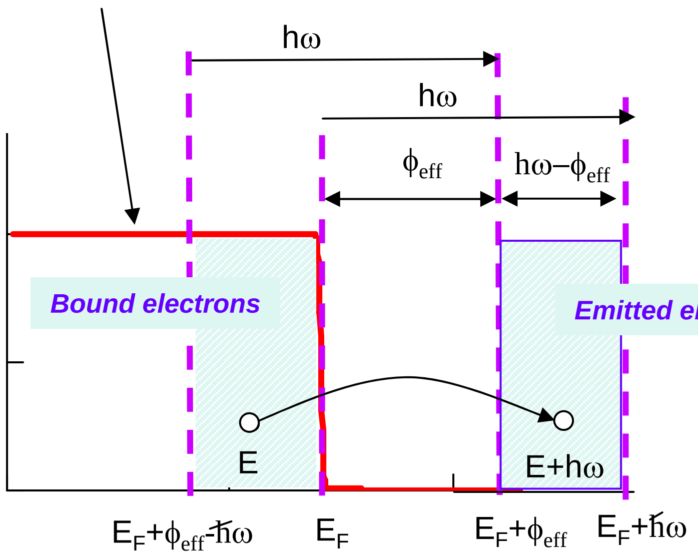
Elements of the Three-Step Photoemission Model

Step 1: Absorption of photon

Fermi-Dirac distribution at 300degK

$$f_{FD}(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}}$$

$$\phi_{eff} = \phi - \phi_{schottky}$$



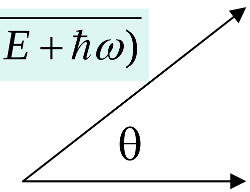
Step 2: Transport to surface

Electrons lose energy by scattering, assume e-e scattering dominates,
F_{e-e} is the probability the electron makes it to the surface without scattering

Step 3: Escape over barrier

Escape criterion: $$\frac{p_{normal}^2}{2m} > E_F + \phi_{eff}$$

$$p_{total} = \sqrt{2m(E + \hbar\omega)}$$



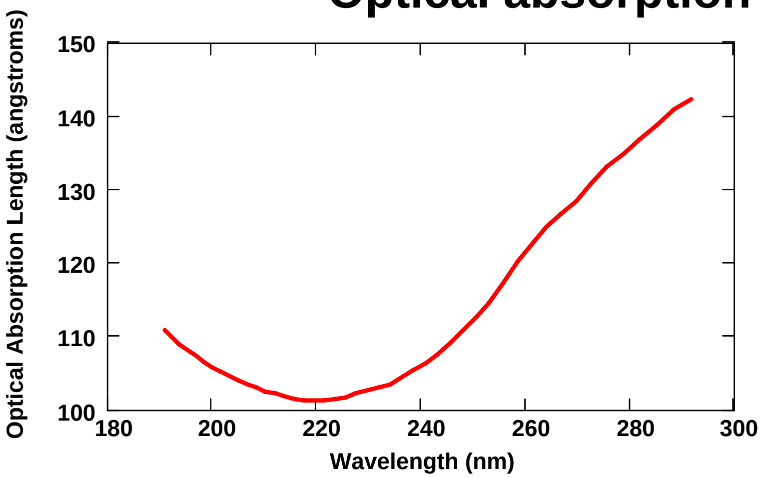
$$p_{normal} = \sqrt{2m(E + \hbar\omega)}\cos\theta$$

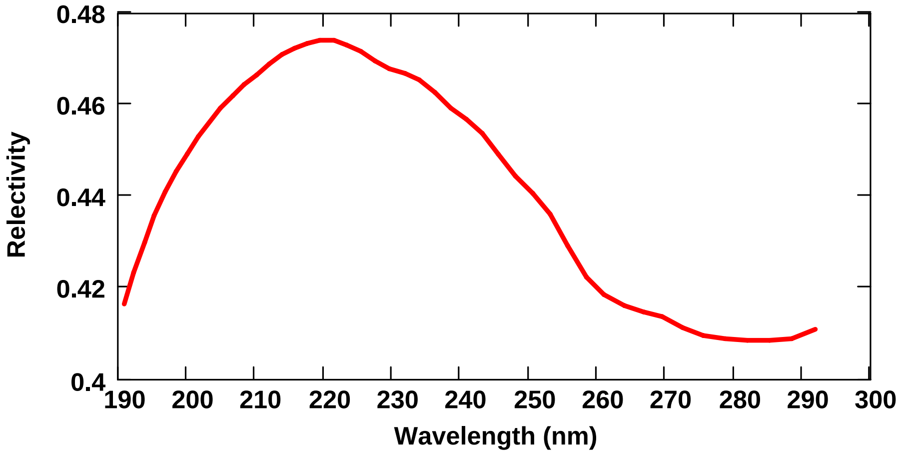
$$\cos\theta_{max} = \frac{p_\perp}{p_{total}} = \sqrt{\frac{E_F + \phi_{eff}}{E + \hbar\omega}}$$

$$QE(\omega) = (1 - R(\omega)) \frac{\int_{E_F+\phi_{eff}-\hbar\omega}^{\infty} dE \; N(E+\hbar\omega)(1 - f_{FD}(E+\hbar\omega))N(E) f_{FD}(E) \int_{\cos\theta_{max}(E)}^{1} d(\cos\theta) F_{e-e}(E,\omega,\theta) \int_0^{2\pi} d\Phi}{\int_{E_F-\hbar\omega}^{\infty} dE \; N(E+\hbar\omega)(1 - f_{FD}(E+\hbar\omega))N(E) f_{FD}(E) \int_{-1}^{1} d(\cos\theta) \int_0^{2\pi} d\Phi}$$

Step 1: Absorption of Photon

Optical absorption length and reflectivity of copper





The optical skin depth depends upon wavelength and is given by,

$$\lambda_{opt} = \frac{\lambda}{4\pi k}$$

where k is the imaginary part of the complex index of refraction,

$$\eta = n + ik$$

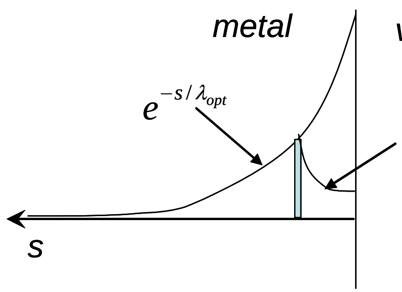
and λ is the free space photon wavelength.

The reflectivity is given by the Fresnel relation in terms of the real part of the index of refraction,

$$\text{Reflectivity} = R(n_1(\omega), n_2(\omega), \theta_i)$$

Step 2: Transport to the Surface

$F_{e\text{-}e}$: Probability electron at depth s, absorbs a photon and escapes without scattering.



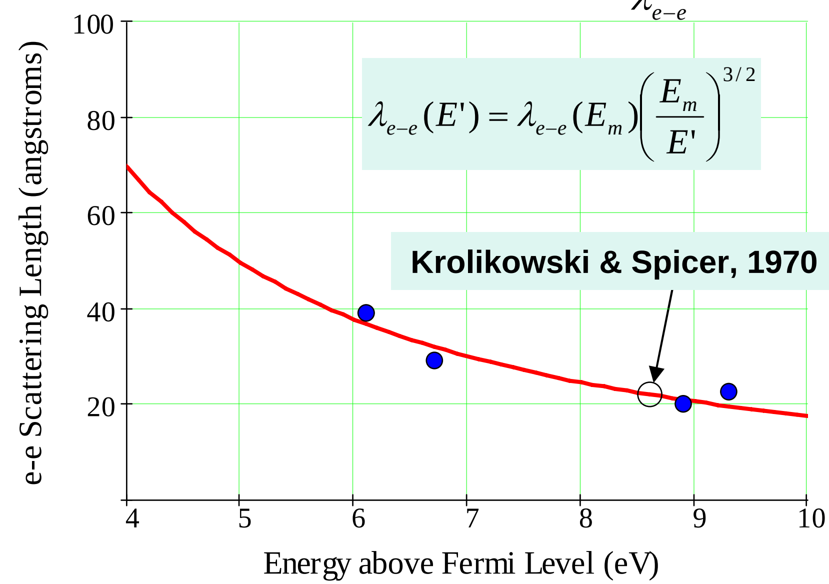
$$e^{-s/\lambda_{opt}} e^{-s/\lambda_{e-e}} = e^{-s\left(\frac{1}{\lambda_{opt}}+\frac{1}{\lambda_{e-e}}\right)} = e^{-s/\lambda_{eff}}$$

$$P_{excited}(s) = \frac{e^{-s\left(\frac{1}{\lambda_{opt}}\right)}}{\int_0^\infty e^{-s/\lambda_{opt}} ds} = \frac{e^{-s/\lambda_{opt}}}{\lambda_{opt}}$$

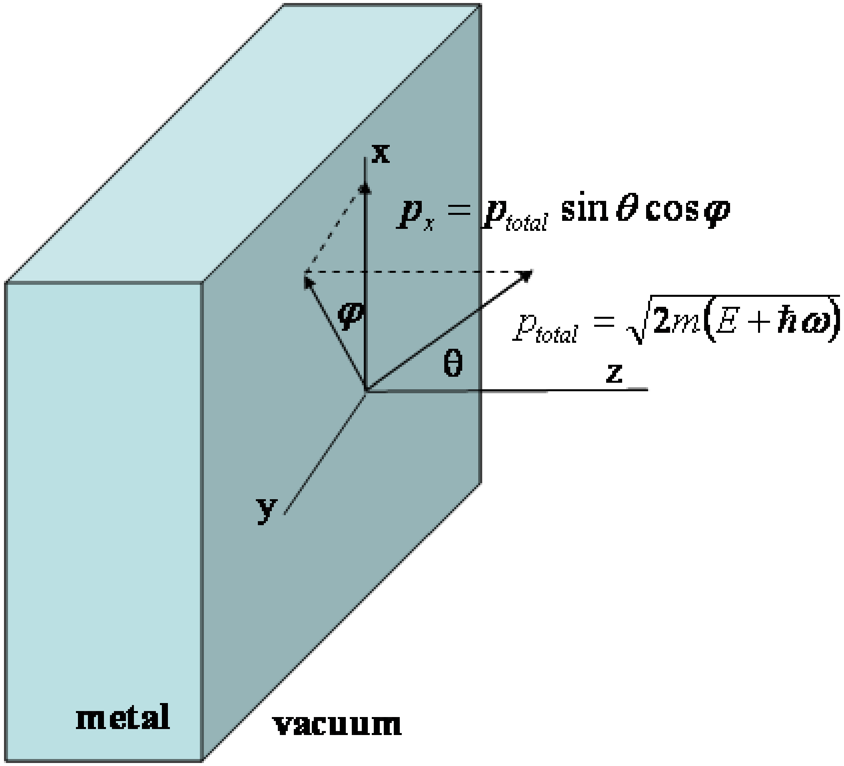
$$f(s) = \frac{1}{\lambda_{opt}} e^{-s\left(\frac{1}{\lambda_{opt}}+\frac{1}{\lambda_{e-e}}\right)} \quad \Longrightarrow \quad F_{e-e} = \int_0^\infty f(s) ds = \frac{1}{1+\frac{\lambda_{opt}}{\lambda_{e-e}}}$$

Assume the electron-electron scattering length can be averaged over energy:

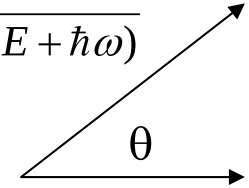
$$\overline{\lambda}_{e-e}(\hbar\omega) = \frac{\int_{\phi_{eff}}^{\hbar\omega} \lambda_{e-e}(E) dE}{\int_{\phi_{eff}}^{\hbar\omega} dE} = \frac{2\lambda_m E_m^{3/2}}{\hbar\omega\sqrt{\phi_{eff}}} \frac{1}{\left(1+\sqrt{\frac{\phi_{eff}}{\hbar\omega}}\right)}$$



Step 3: Escape Over the Barrier



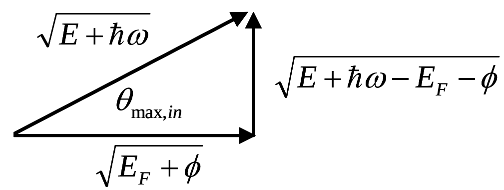
$$p_{total} = \sqrt{2m(E+\hbar\omega)}$$



$$p_{normal} = \sqrt{2m(E+\hbar\omega)}\cos\theta$$

Escape criterion: $$\frac{p_{normal}^2}{2m} > E_F + \phi_{eff}$$

$$\cos\theta_{\max} = \frac{p_{normal}}{p_{total}} = \sqrt{\frac{E_F + \phi_{eff}}{E+\hbar\omega}}$$

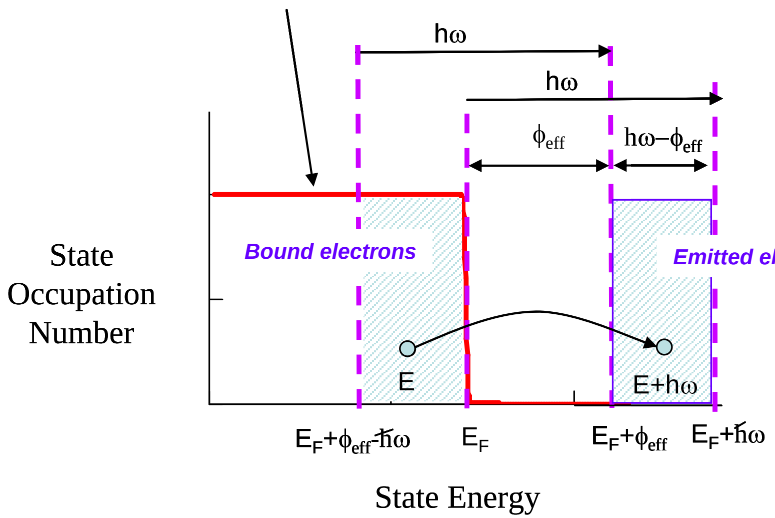


While photoemission is regarded quantum mechanical effect due to quantization of photons, emission itself is classical. I.e., electrons do not tunnel through barrier, but classically escape over it.

Derivation of QE

$$QE(\omega)=(1-R(\omega))\frac{\int_{E_F+\phi_{eff}-\hbar\omega}^{\infty} dE\;\; N(E+\hbar\omega)(1-f_{FD}(E+\hbar\omega))N(E)f_{FD}(E)\int_{\cos\theta_{\max}(E)}^{1} d(\cos\theta)F_{e-e}(E,\omega,\theta)\int_0^{2\pi} d\Phi}{\int_{E_F-\hbar\omega}^{\infty} dE\;\; N(E+\hbar\omega)(1-f_{FD}(E+\hbar\omega))N(E)f_{FD}(E)\int_{-1}^{1} d(\cos\theta)\int_0^{2\pi} d\Phi}$$

$$f_{FD}(E)=\frac{1}{1+e^{(E-E_F)/k_BT}}$$



Approximate F-D with step function since $k_BT<<E_F$:

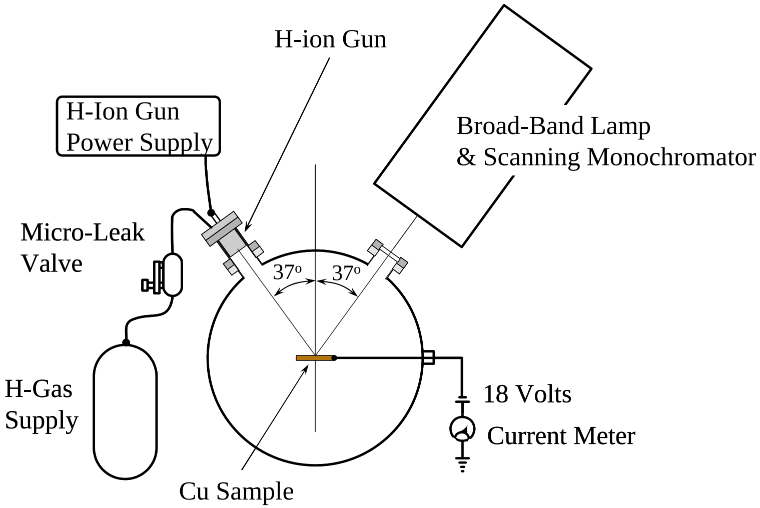
$$QE(\omega)=\left(1-R(\omega)\right)F_{e-e}(\omega)\frac{\int_{E_F+\phi_{eff}-\hbar\omega}^{E_F} dE\int_{\sqrt{\frac{E_f+\phi_{eff}}{E+\hbar\omega}}}^{1} d(\cos\theta)\int_0^{2\pi} d\Phi}{\int_{E_F-\hbar\omega}^{E_F} dE\int_{-1}^{1} d(\cos\theta)\int_0^{2\pi} d\Phi}$$

The QE is then given by:

$$QE(\omega)=\frac{1-R(\omega)}{1+\frac{\lambda_{opt}(\omega)}{2\lambda_{e-e}(E_m)}\frac{\hbar\omega\sqrt{\phi_{eff}}}{E_m^{3/2}}\left(1+\sqrt{\frac{\phi_{eff}}{\hbar\omega}}\right)}\frac{(E_F+\hbar\omega)}{2\hbar\omega}\left[1-2\sqrt{\frac{E_F+\phi_{eff}}{E_F+\hbar\omega}}\right]^2$$

- D. H. Dowell, K.K. King, R.E Kirby and J.F. Schmerge, "In situ cleaning of metal cathodes using a hydrogen beam," PRST-AB 9, 063502 (2006)

Comparison of Theory and Experiment
QE vs. Wavelength



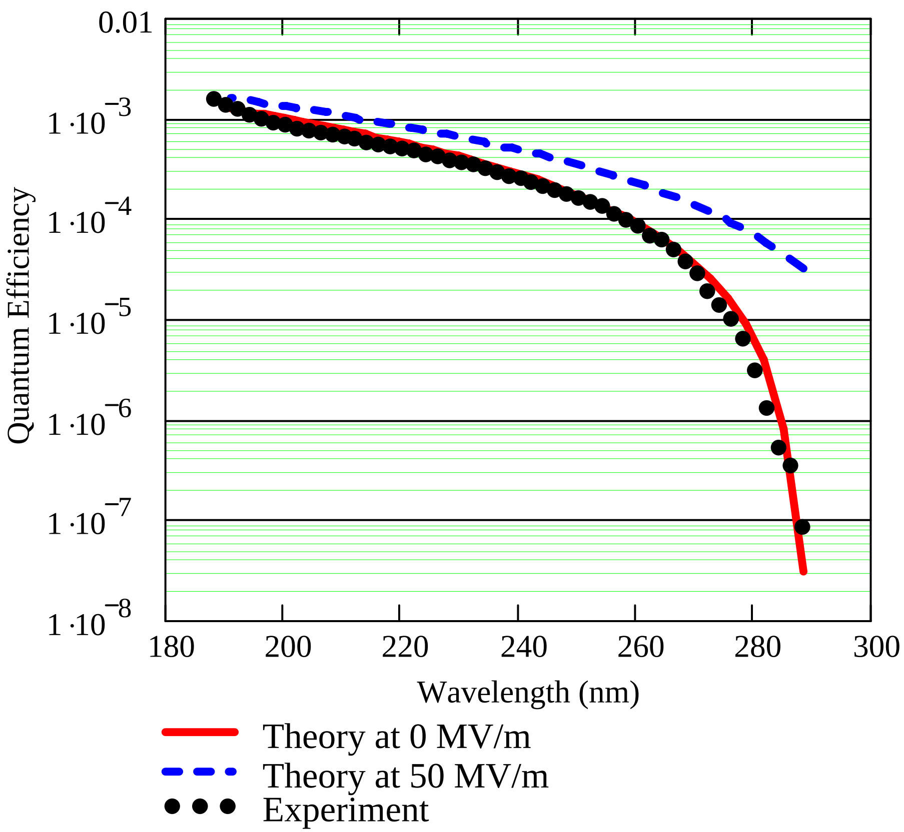


Photo-Electric Emittance (1)

- The mean square of the transverse momentum is related to the electron distribution function, *g(E,θ,φ),* just inside the cathode surface,

$$\langle p_{tot}^2 \rangle = \frac{\int\int\int g(E,\theta,\varphi)p_x^2 dEd(cos\theta)d\varphi}{\int\int\int g(E,\theta,\varphi)dEd(cos\theta)d\varphi}$$

- The *g*-function and the integration limits depend upon the emission processes. We assume for the three-step photo-emission model that *g* depends only on energy,

$$g_{photo} = (1 - f_{FD}(E + \hbar\omega))f_{FD}(E)$$

Photo-Electric Emittance (2)

- Since the Fermi-Dirac function, f_{FD}, at temperatures near ambient is well-represented by a Heaviside-step function which then determines the limits on the energy integration. The θ–integration ranges from zero to θ_{max}. Since the transverse moment, p_x, is

$$p_x = \sqrt{2m(E+\hbar\omega)} sin\theta cos\varphi$$

- The mean square of the x-momentum becomes,

$$\langle p_{tot}^2 \rangle = \frac{2m \int_{E_F+\phi_{eff}-\hbar\omega}^{E_F} dE \int_{\sqrt{\frac{E_F+\phi_{eff}}{E+\hbar\omega}}}^{1} d(cos\theta) \int_0^{2\pi} d\varphi (E+\hbar\omega) sin^2\theta cos^2\varphi}{\int dE \int d(cos\theta) \int d\varphi}$$

- Performing these integrals gives the photo-electric normalized emittance

$$\epsilon_{photo} = \sigma_x \sqrt{\frac{\hbar\omega - \phi_{eff}}{3mc^2}}$$

Photo-Electric Emittance (3)

- Normalized divergence vs. photon energy for various applied fields

$$\Delta_{photo} = \beta\gamma\sigma_{x'}^{photo} = \sqrt{\frac{\hbar\omega - \phi_{eff}}{3mc^2}}$$

$$\epsilon_{photo} = \sigma_x \sqrt{\frac{\hbar\omega - \phi_{eff}}{3mc^2}}$$

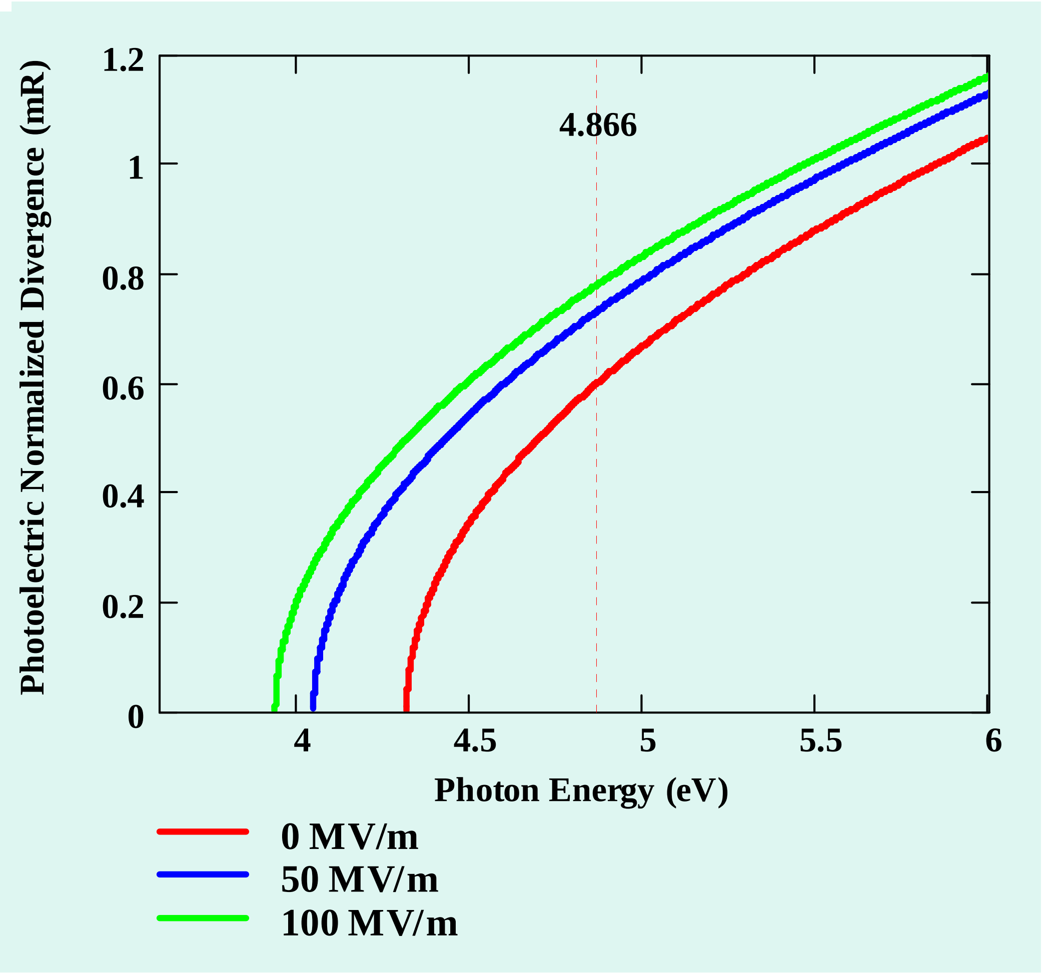
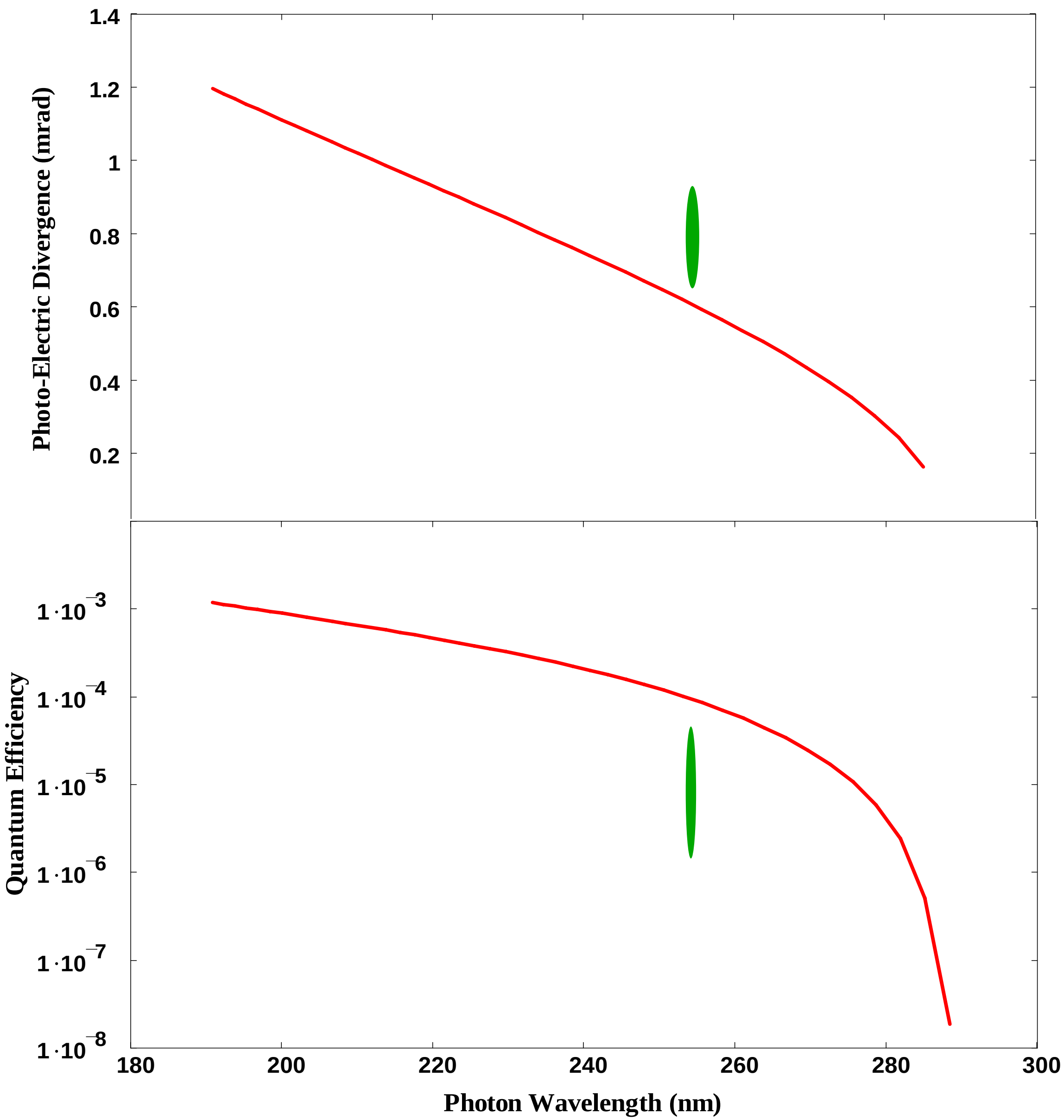


Photo-Electric Emittance & QE



The photo-electric normalized divergence (top, red) and the quantum efficiency (bottom, red) vs. photon wavelength for copper. The green ellipses show the range of QE and cathode emittance measured during commissioning of the LCLS injector.

Field Emission (1)

- Field emission occurs when electrons tunnel through the barrier potential under the influence of very high fields of 10^9 V/m or more. Since emission is by tunneling the effect is purely quantum mechanical and requires an extremely high electric field to lower the barrier enough for useful emission.

$$j = \int n(E_x, T) D(E_x, E_0) dE_x$$

- where the supply function, $n(E_x,T)$, is the flux of electrons incident upon the barrier with energies between E_x and $E_x + dE_x$. The barrier is same as that shown earlier and is determined by the work function, the image charge and the applied electric field, E_0. The transmission of electrons through this barrier is given by the transparency function, $D(E_x,E_0)$.

Field Emission (2)

- The transparency function was solved by Nordheim for the barrier produced by the image charge and the applied field (Schottky potential),

$$\phi_{Schottky}(x) = -\frac{e^2}{16\pi\epsilon_0 x} - eE_0 x$$

- The result is

$$D(E_x, E_0) = \exp\left[\frac{-8\pi\sqrt{2m}}{3he}\frac{E_x^{3/2}}{E_0}\theta\left(\frac{\sqrt{e^3 E_0}}{\phi_{work}}\right)\right]$$

- $\theta(y)$ is the Nordheim function which to a good approximation is given by

$$\theta(y) = 1 - 0.142y - 0.855y^2$$

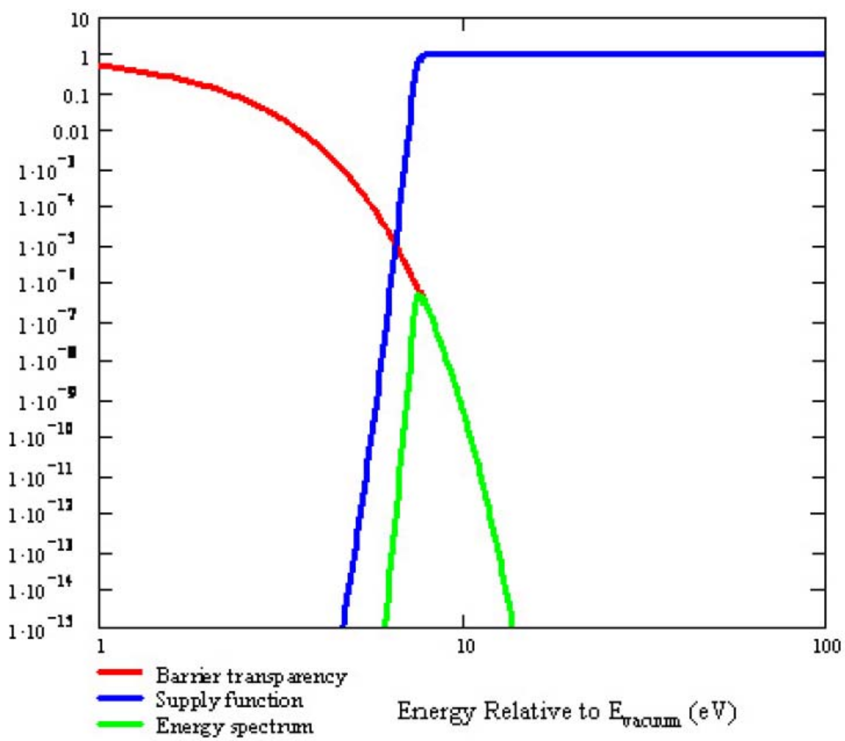
Field Emission (3)

- The supply function for a Fermi-Dirac electron gas was also derived by Nordheim,

$$n(E_x, T) = \frac{4\pi m k_B T}{h^3} \ln\left(1 + e^{\frac{E_x - E_F}{k_B T}}\right)$$

- Combining the supply and transparency functions gives the electron energy spectrum,

$$N_{field}(E_x, E_0, T) = n(E_x, T) D(E_x, E_0)$$

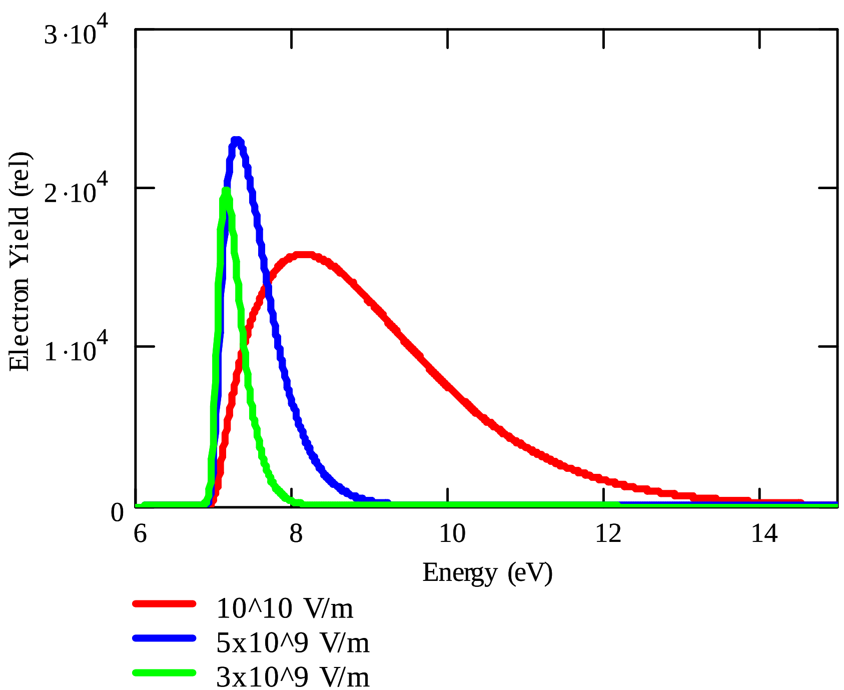


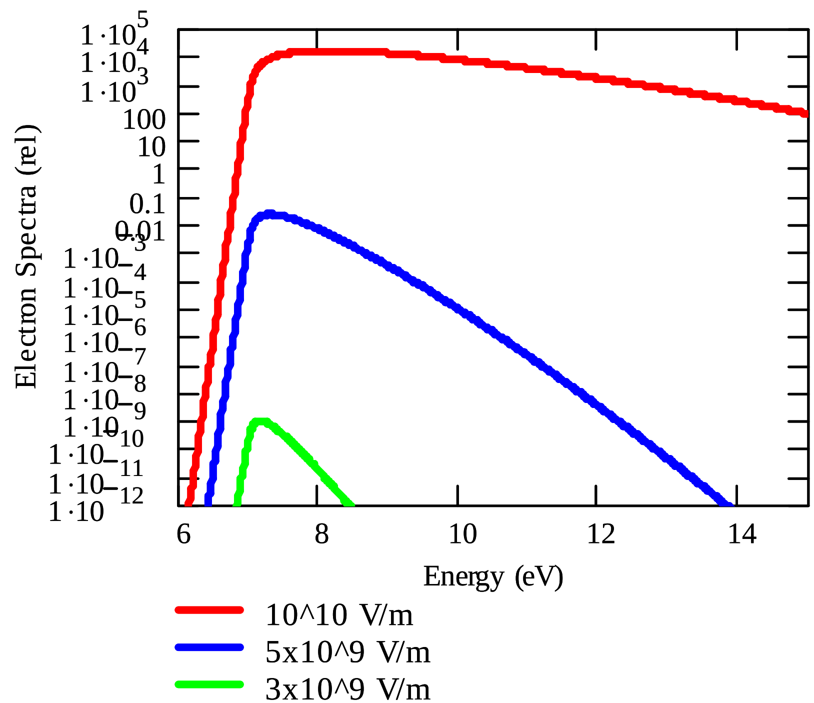
"Field Emission in Vacuum Microelectronics," G. Fursey, Kluwer Academic/Plenum, 2005

Field Emission (4)

- Electron spectra for field emission electrons for various applied fields. Left: Electron emission spectra plotted with a linear vertical scale and with arbitrarily normalization to illustrate the spectral shapes.

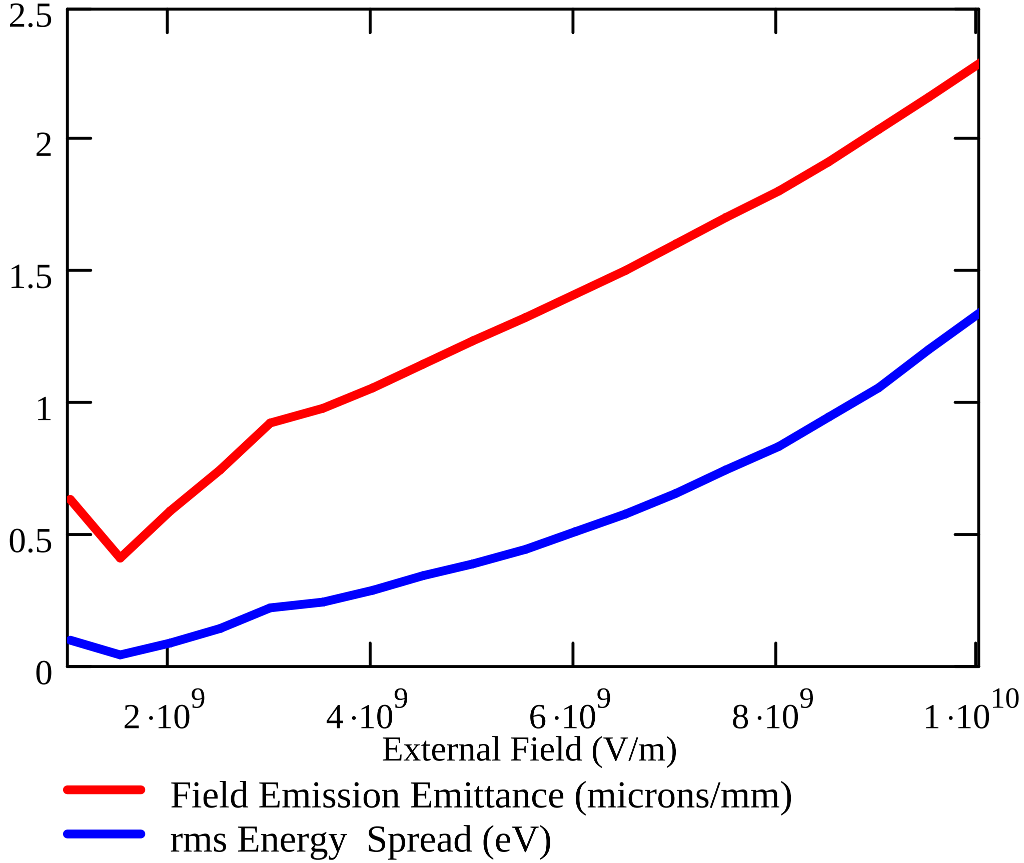
  Right: The spectral yields plotted logarithmically to illustrate the strong dependence of yield and shape upon applied field.





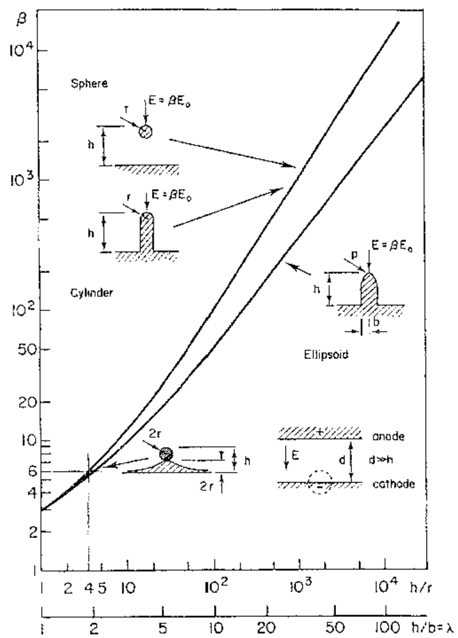
Field Emission Emittance

- Armed with the energy spectra the rms energy spread and the field emission emittance are numerically computed for external fields between 10^9 and 10^{10} Volts/m. (Solved numerically.)



Field Enhancement Factor, $E = \beta E_0$

- In field emission the electron yield is exponentially sensitive to the external field and any significant current requires fields in excess of 10^9 V/m. Such high fields are difficult to achieve but are possible using pulsed high voltages and/or field-enhancing, sharp emitters.



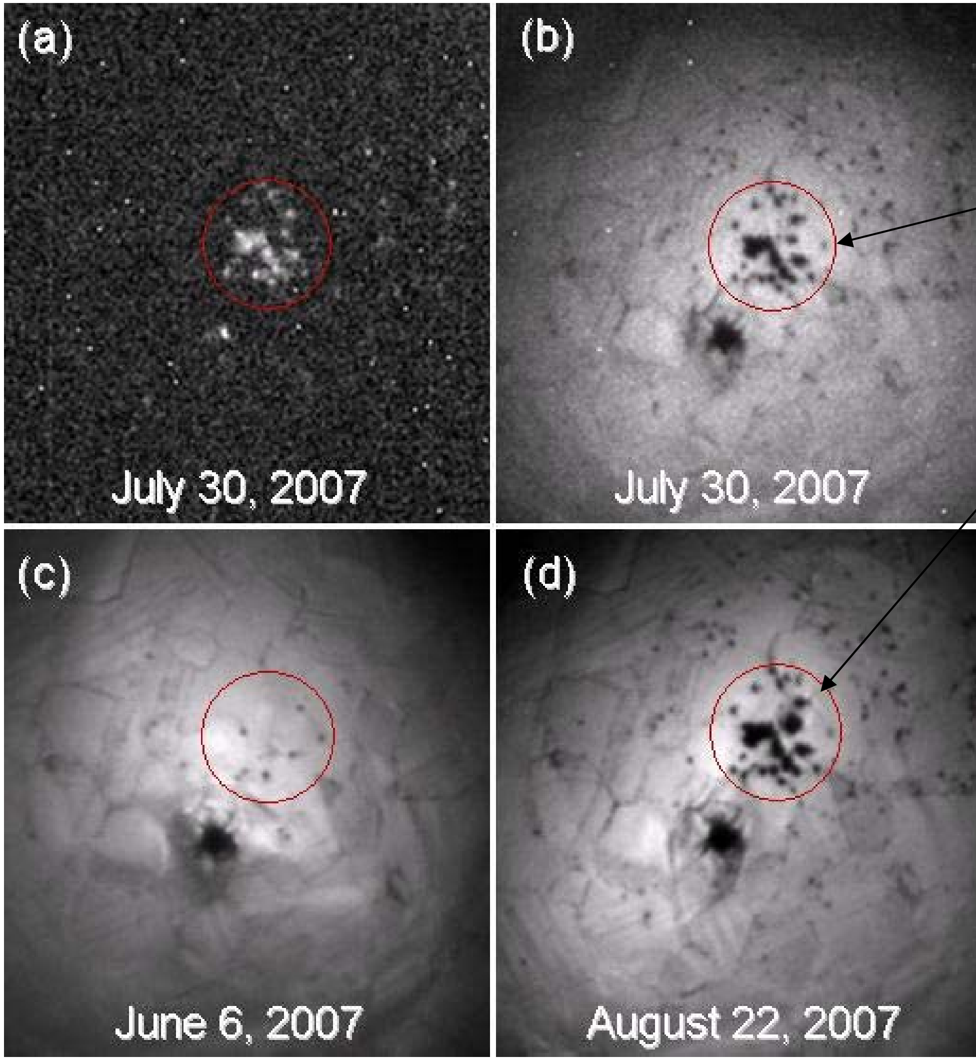
A collated representation of the field enhancement factors β associated with various idealised microprotrusion geometries. (From Rohrbach [31], with permission.)

"High Voltage Vacuum Insulation, Basic Concepts and Technological Practice," Ed. Rod Latham, Academic Press 1995

- The space charge limit and cathode emittance

What A Real Cathode Looks Like!

LCLS gun cathode after 5 months of operation



1.4 mm diameter Illuminated by drive laser

Lecture 2 Summary

- This lecture derived the cathode emittances for the three emission processes: thermionic, photo-electric and field emission. Rather than using the term, thermal emittance, we prefer to use the general term of cathode or intrinsic emittance for any emission process. And instead define the intrinsic emittance for each of the three emission processes. The intrinsic emittance for thermionic emission is approximately 0.3 microns/mm for a cathode temperature of 2500 degK. The photo-electric emittance for a copper cathode ranges between 0.5 to 1 micron/mm depending upon the photon wavelength, and the emittance was shown to be proportional to the quantum efficiency. The field-emission emittance is found to vary between 0.5 to 2 microns/mm for fields from 10^9 to 10^{10} V/m, and hence has larger emittance for the same source size than the other two processes.